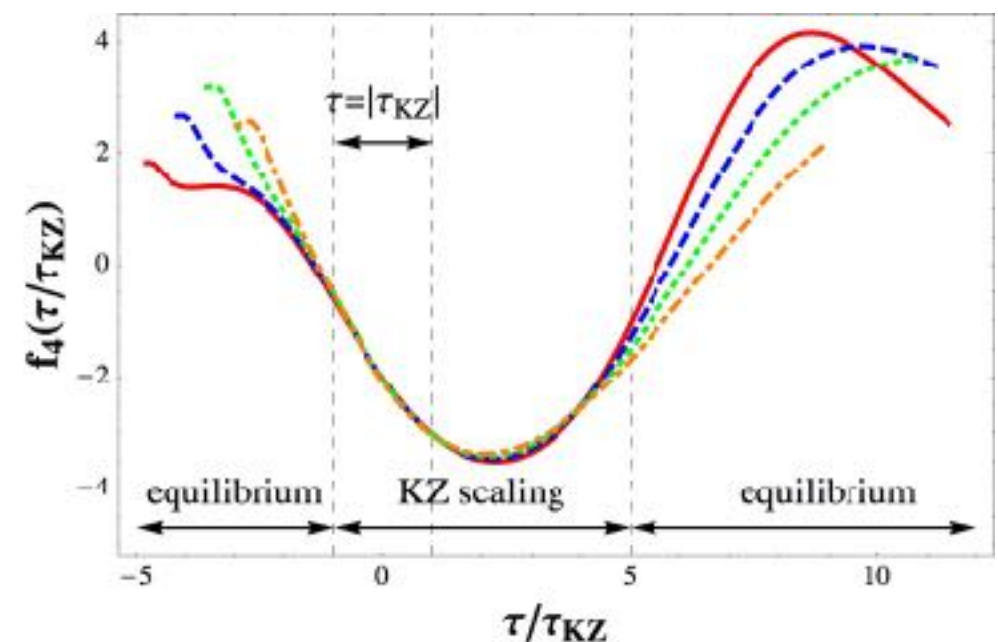
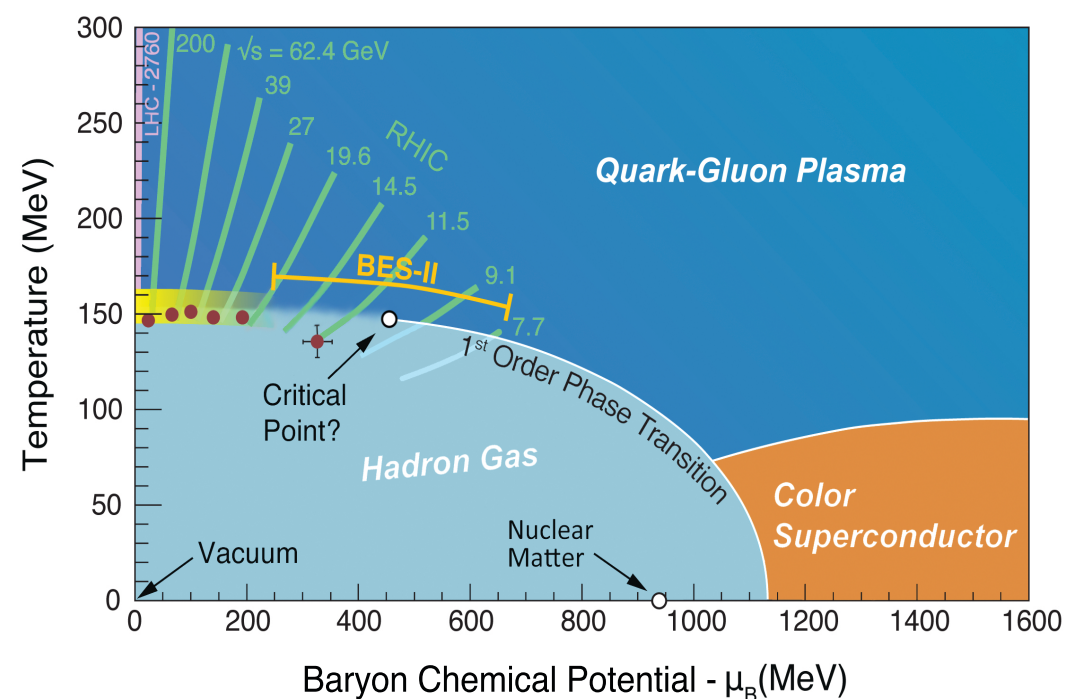


# Kibble-Zurek dynamics, hydrodynamics and turbulent cascade



(PRL, Editors' suggestion, 16')

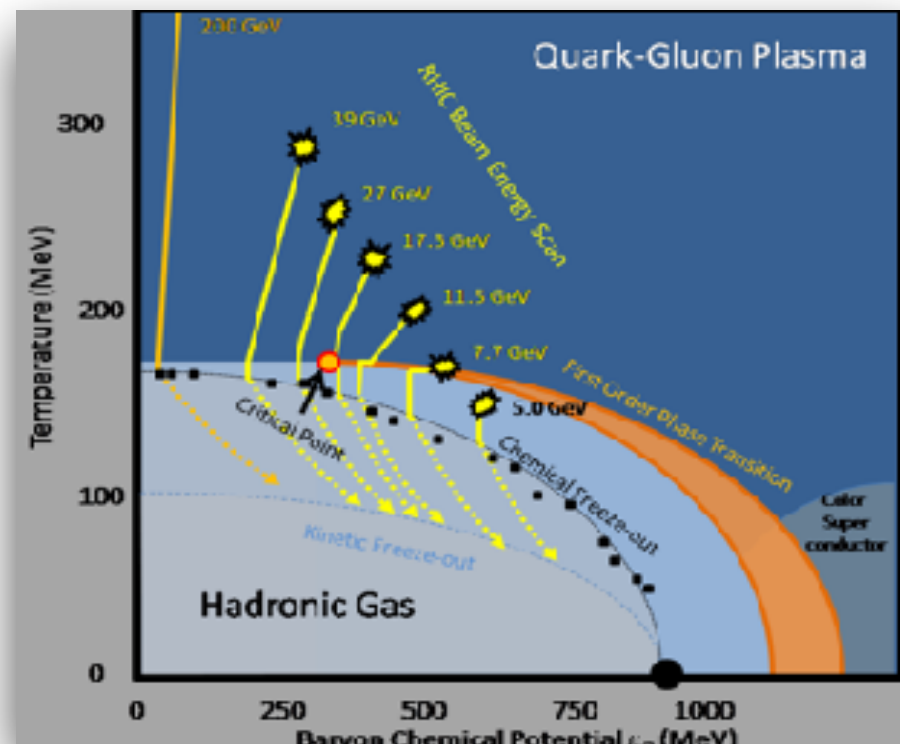
**BEST**  
COLLABORATION

Yi Yin



# Outline

- Motivation: off-equilibrium evolution near the critical point.
- Kibble-Zurek dynamics and search for QCD critical point.  
(S. Mukherjee, R. Venugopalan and YY, PRL, Editors' suggestion, 16')
- Generalization to Critical Hydrodynamics.  
(D. Teaney, F. Yan, Y. Akamatsu, YY, in progress)
- Outlook: connection to turbulent cascade



- Beam energy scan program: “quench” across QCD phase diagram.

$$(\sqrt{s}, b) \longrightarrow (\mu_B(\tau), T(\tau))$$

- **Key question:** what do we expect to see experimentally?
- In a broad context: off-equilibrium evolution near a critical system.

# The manifestation of criticality

- Correlation length  $\xi$  of the critical mode  $\sigma$  grows.
- Fluctuations: sensitive to the growth of  $\xi$ .  $\kappa_n \sim \langle (\delta\sigma)^n \rangle$

$$\kappa_2^{\text{eq}} \sim \xi_{\text{eq}}^2 \quad \kappa_3^{\text{eq}} \sim \xi_{\text{eq}}^{9/2} \quad \kappa_4^{\text{eq}} \sim \xi_{\text{eq}}^7$$

3d Ising model



QCD critical point

(Berges, Rajagopal, 98)

Magnetization  
(critical mode)  $\sigma$



A mixture of baryon density  $n_B$   
energy density  $\varepsilon$  and chiral  
condensate.

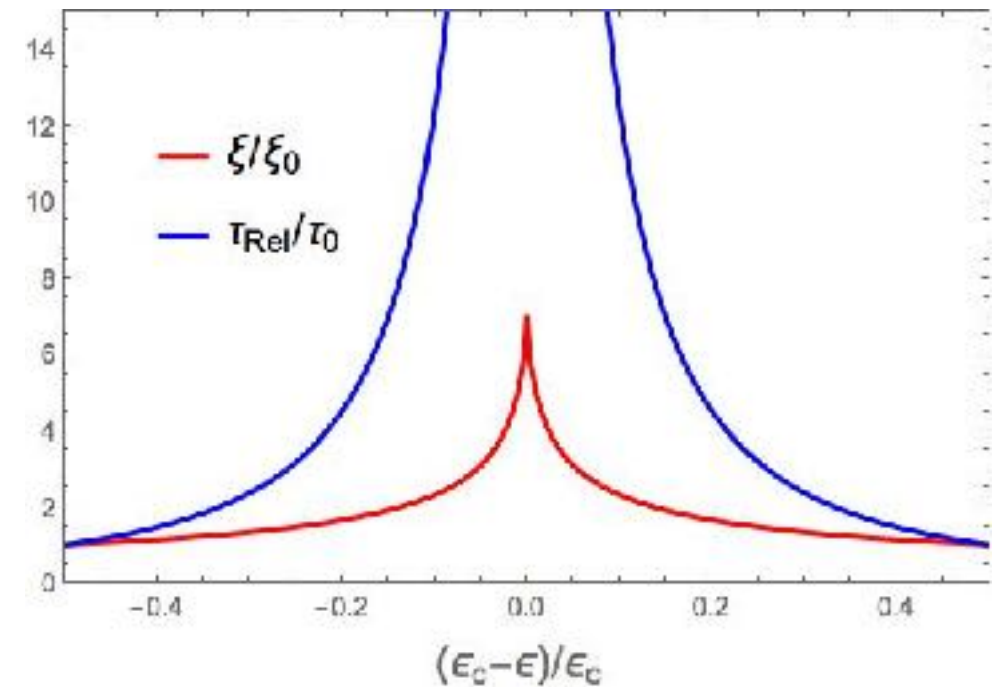
Observables:  $\langle (\delta N_B)^n \rangle \sim \kappa_n \sim \langle (\delta\sigma)^n \rangle$

- Non-Gaussian fluctuations: universal pattern in sign.

- Critical mode is off-equilibrium !

$$\tau_\sigma \sim \xi_{\text{eq}}^z, \quad z \approx 3$$

(Son, Stephanov, 2008)



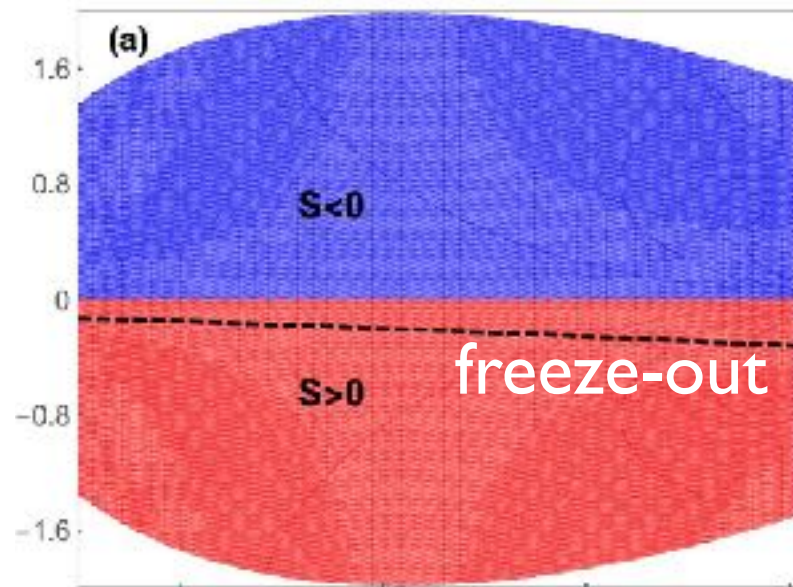
- Off-equil. Gaussian cumulants : **Berdinkov-Rajagopal, 99'**
- Off-equil. Gaussian and non-Gaussian cumulants.  
(S. Mukherjee, R. Venugopalan and YY, 15')

(see M. Nahrgang, QM 15' proceedings)

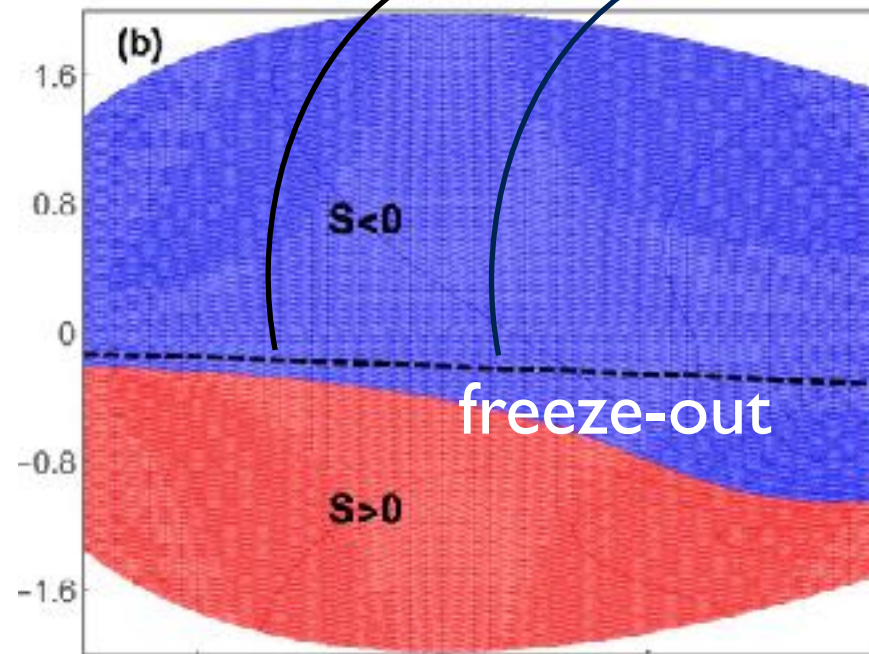


## Equilibrium

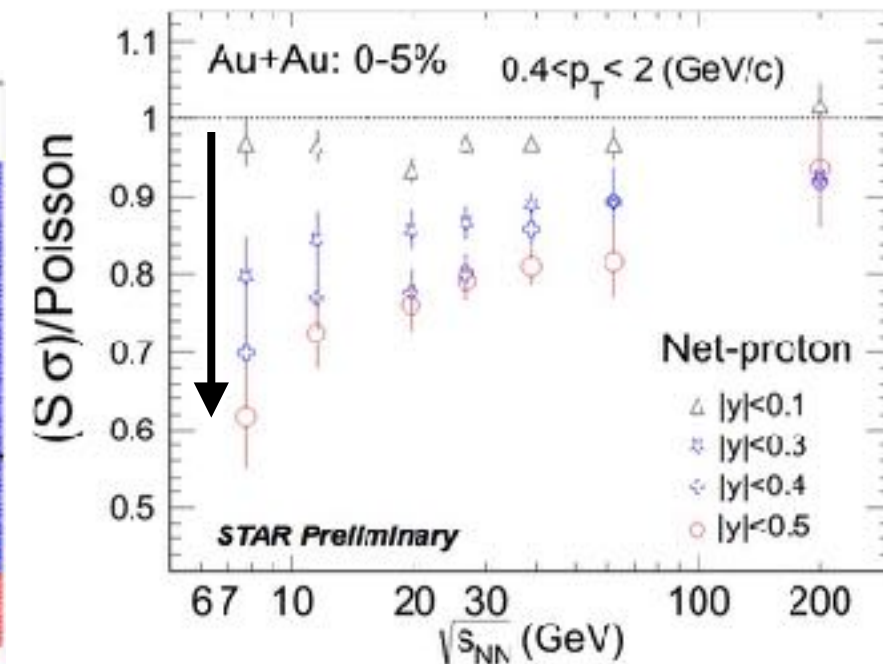
## non-equilibrium



Decreasing  $\sqrt{s}$



Decreasing  $\sqrt{s}$



Decreasing  $\sqrt{s}$

(S. Mukherjee, R. Venugopalan and YY, PRC, 2015)

- Non-equilibrium evolution can be qualitatively different from the naive equilibrium expectations.

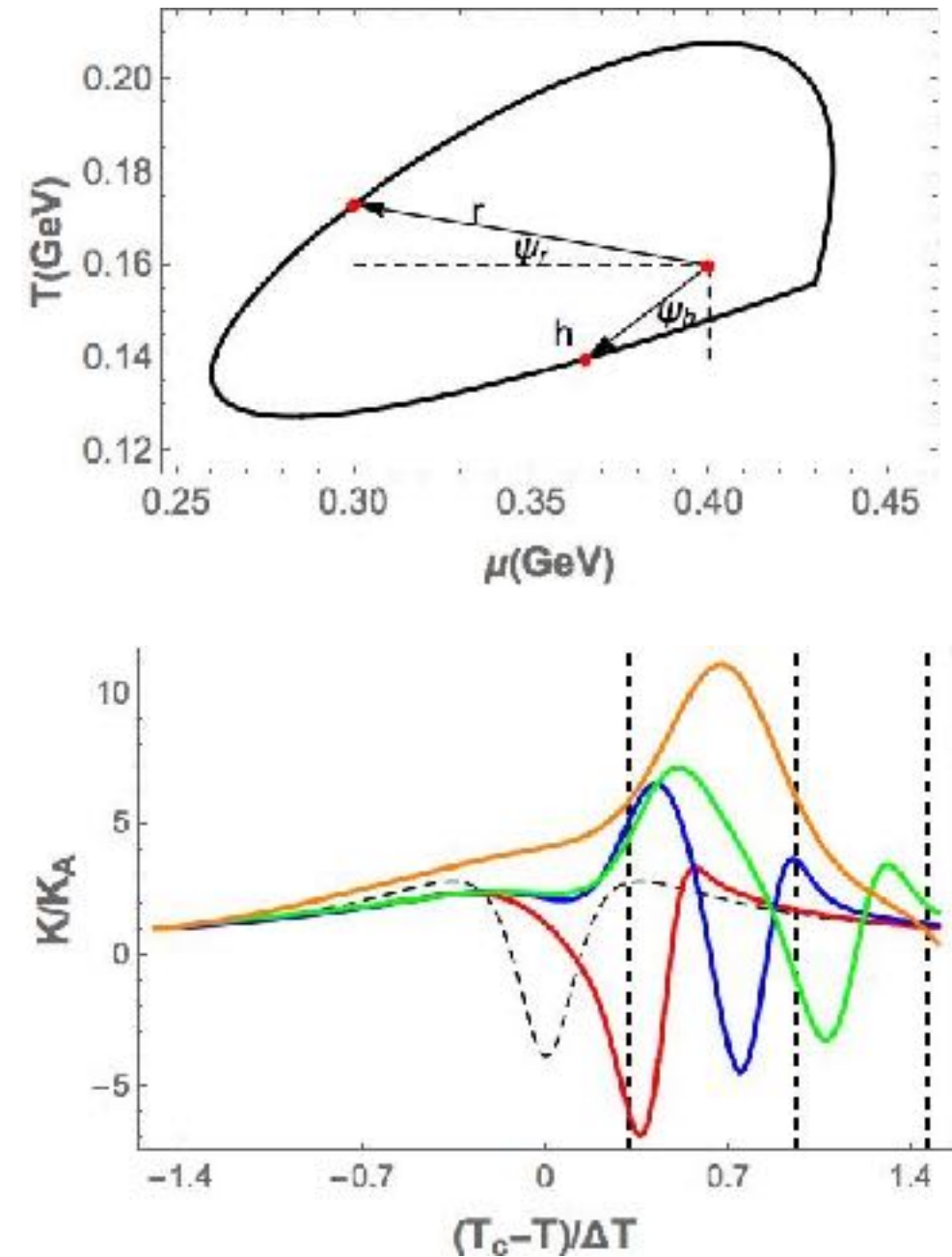
*Equilibrium=Forgetting; Off-equil.=Memory .*

Memory implies *complexity*!

- Evolution depends on many non-universal inputs:
- mapping, location of critical point, width of critical regime.
- Trajectories in phase diagram
- relaxation rate and expansion rate.
- The non-equilibrium cumulants look complicated.

?

“此情可待成追忆，  
只是当时已惘然。”



- Is universality lost in complexity ?
- Is there a simple way to understand what has been “memorized”?
- Answers to those questions are connected by:

*Kibble-Zurek dynamics*



Topological defects in cosmological phase transitions. (T.W. Kibble, *Physics Reports* 67, 183 (1980) )

Generalized to vortex generation in superfluids. ( W. H. Zurek, “Cosmological experiments in superfluid helium?”, *Nature* 317, 505 (1985) )

# The Kibble-Zurek Problem: Universality and the Scaling Limit

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(Dated: September 20, 2012)

PRL 109, 015701 (2012)

PHYSICAL REVIEW LETTERS

week ending  
6 JULY 2012

## Nonequilibrium Dynamic Critical Scaling of the Quantum Ising Chain

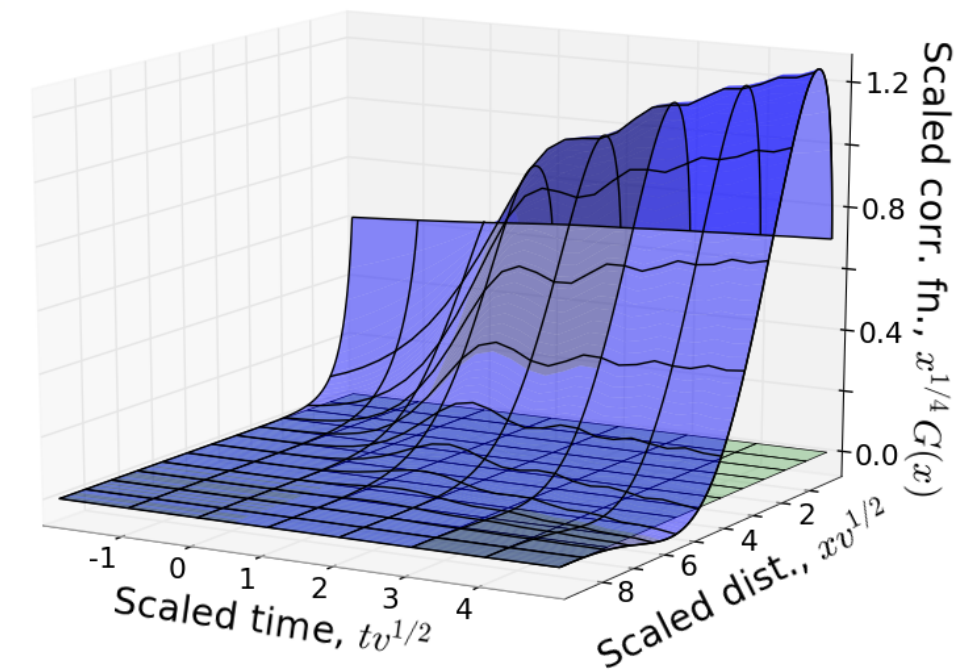
Michael Kolodrubetz,<sup>1</sup> Bryan K. Clark,<sup>1,2</sup> and David A. Huse<sup>1,2</sup>

<sup>1</sup>*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

<sup>2</sup>*Princeton Center for Theoretical Science, Princeton University, Princeton, New Jersey 08544, USA*

(Received 2 February 2012; published 2 July 2012)

We solve for the time-dependent finite-size scaling functions of the one-dimensional transverse-field Ising chain during a linear-in-time ramp of the field through the quantum critical point. We then simulate Mott-insulating bosons in a tilted potential, an experimentally studied system in the same equilibrium universality class, and demonstrate that universality holds for the dynamics as well. We find qualitatively



week ending  
26 FEBRUARY 2016

PRL 116, 080601 (2016)

PHYSICAL REVIEW LETTERS

## Universality in the Dynamics of Second-Order Phase Transitions

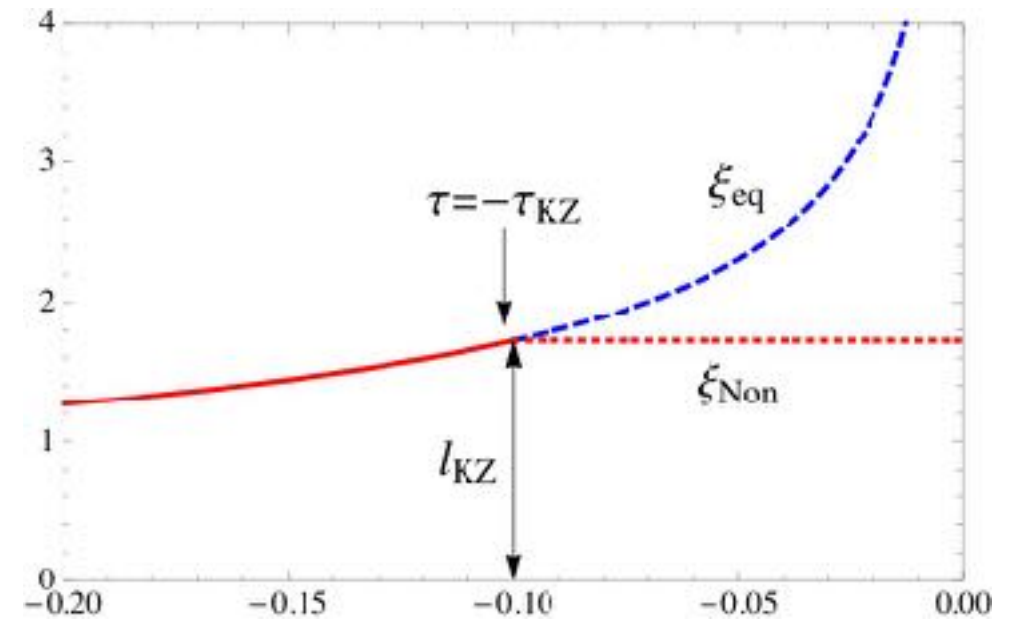
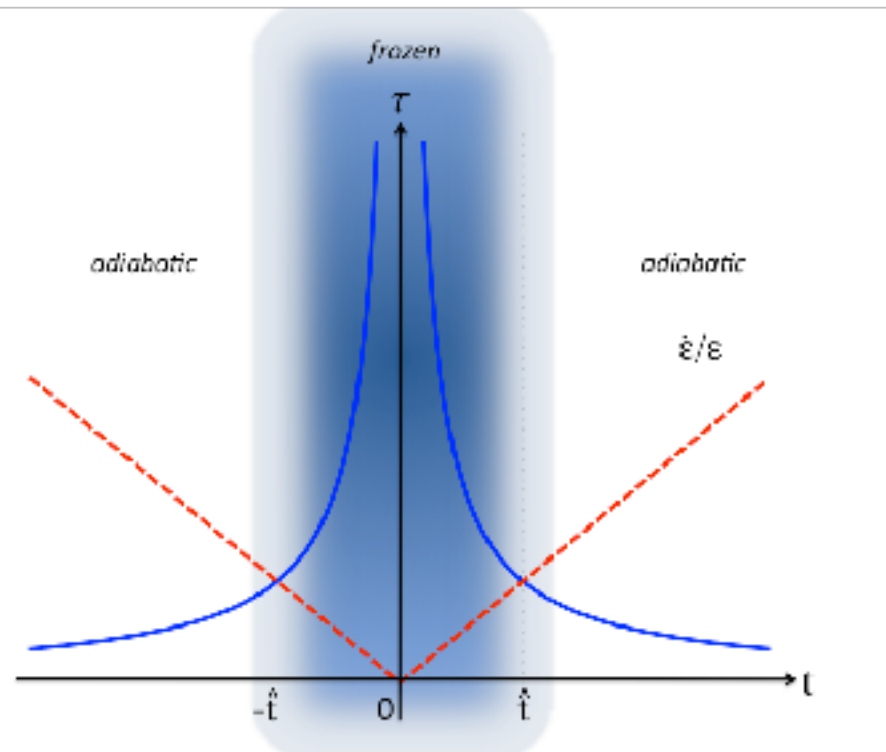
G. Nikoghosyan,<sup>1,2</sup> R. Nigmatullin,<sup>3</sup> and M. B. Plenio<sup>1</sup>

<sup>1</sup>*Institut für Theoretische Physik, Albert-Einstein Allee 11, Universität Ulm, 89069 Ulm, Germany*

<sup>2</sup>*Institute of Physical Research, 378410 Ashtarak-2, Armenia*

<sup>3</sup>*Department of Materials, University of Oxford, Oxford OX1 3PH, United Kingdom*

(Received 18 November 2013; revised manuscript received 10 February 2015; published 26 February 2016)



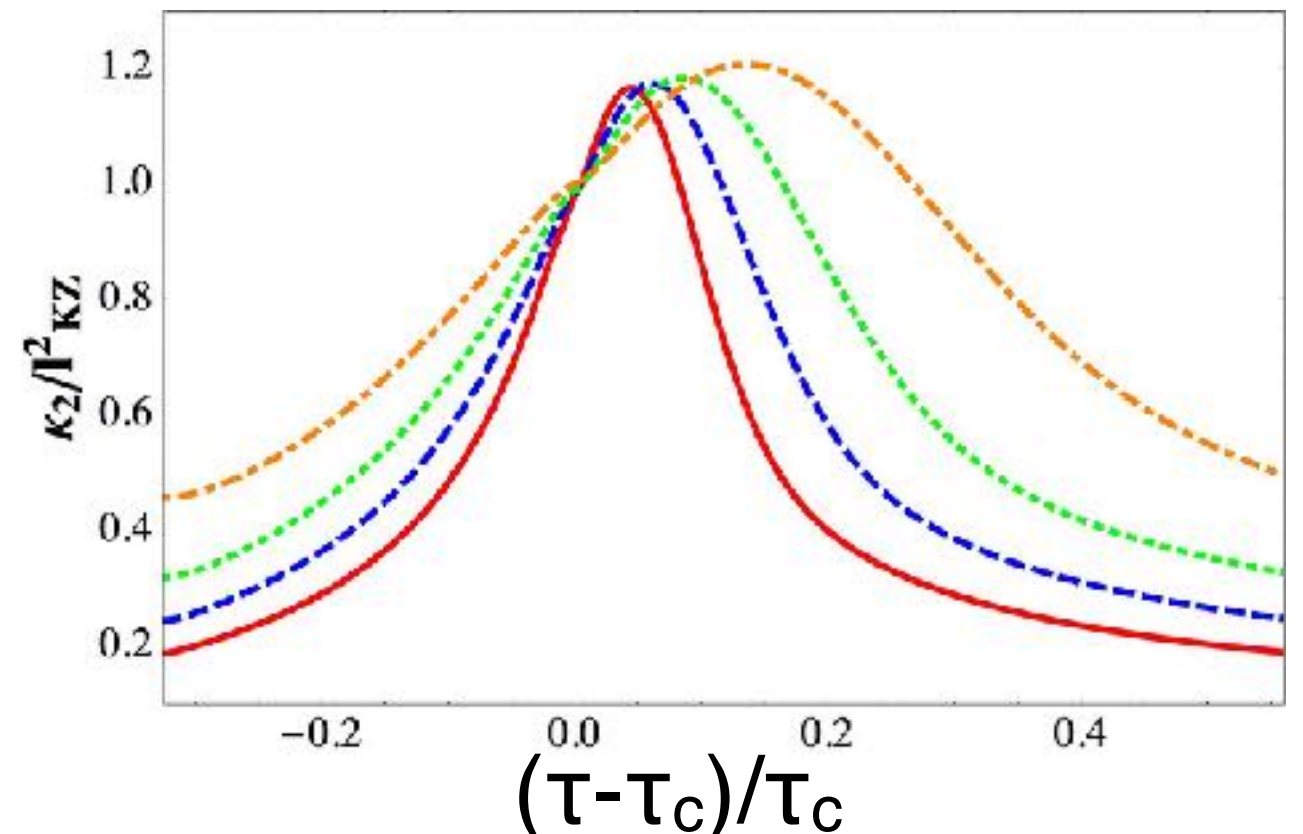
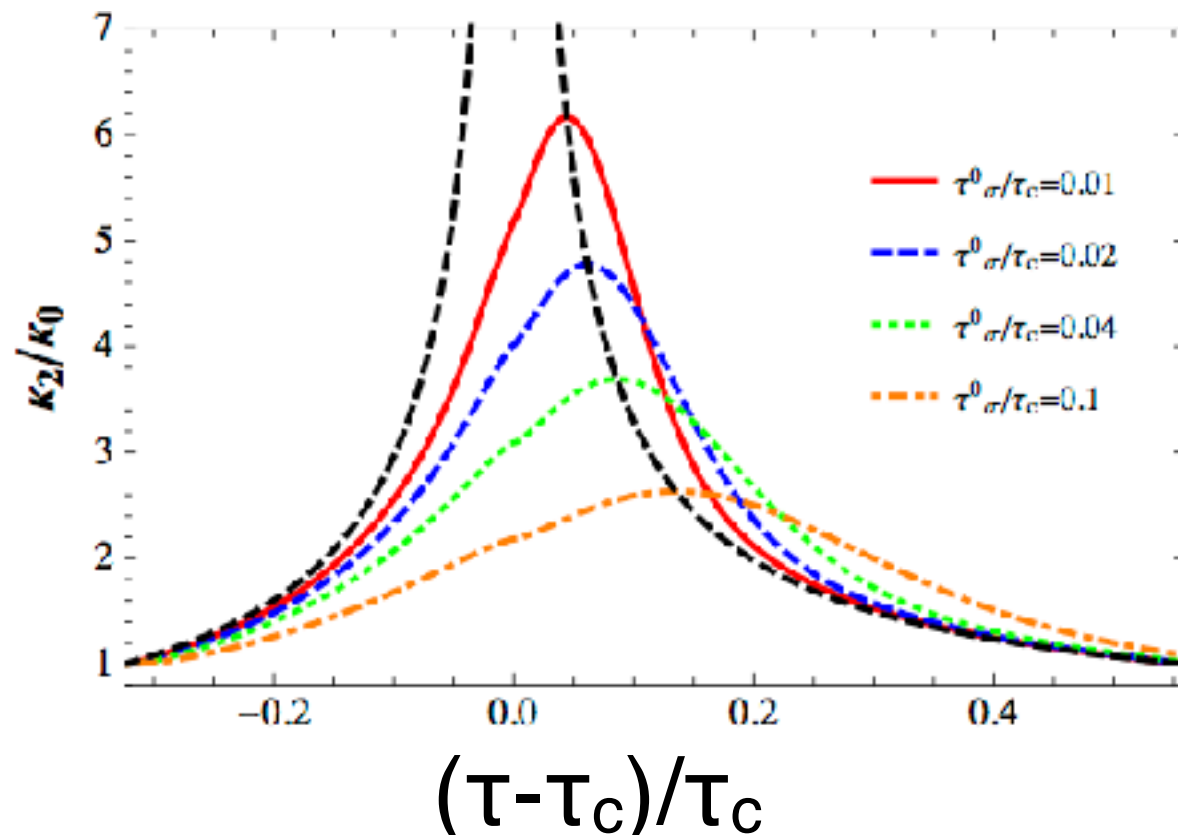
arXiv:1310.1600

- Critical slowing down: relaxation time grows and becomes shorter than the expansion (“quench”) time.
- The correlation length is frozen at the value when:  
**Quench (expansion) time = Relaxation time**
- Kibble-Zurek dynamics:

$$l_{\text{KZ}} = \xi_{\text{eq}}(\tau_{\text{KZ}}) \quad \text{Magnitude (1980s).}$$

$\tau_{\text{KZ}}$

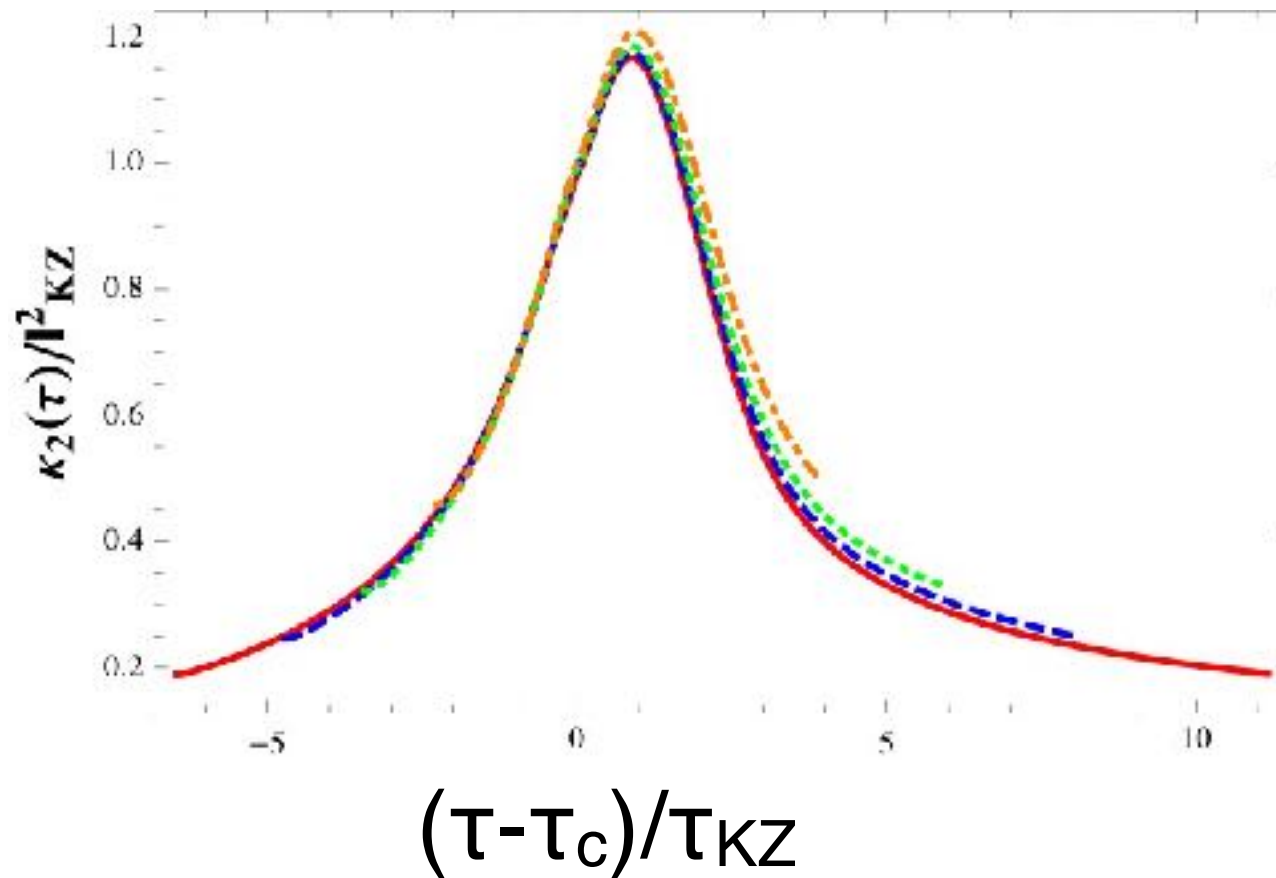
*Time evolution (2010s).*



An illustrative example: non-equilibrium evolution of correlation length (Berdnikov-Rajagopal model revisited).

- Rescale Gaussian cumulants by  $(l_{kz})^2$ .
- Scaling with length is not enough!
- A step forward: rescale time by  $\tau_{kz}$  !





*“you can hide but you can not run.”*

- Off-equilibrium scaling function !

$$\kappa_2(\tau; \Gamma) \sim l_{KZ}^2(\Gamma) \underbrace{f_2(\tau / \tau_{KZ}(\Gamma))}_{\text{Universal}}$$

( $\Gamma$  : non-universal inputs)

*New physics in an old paper!*

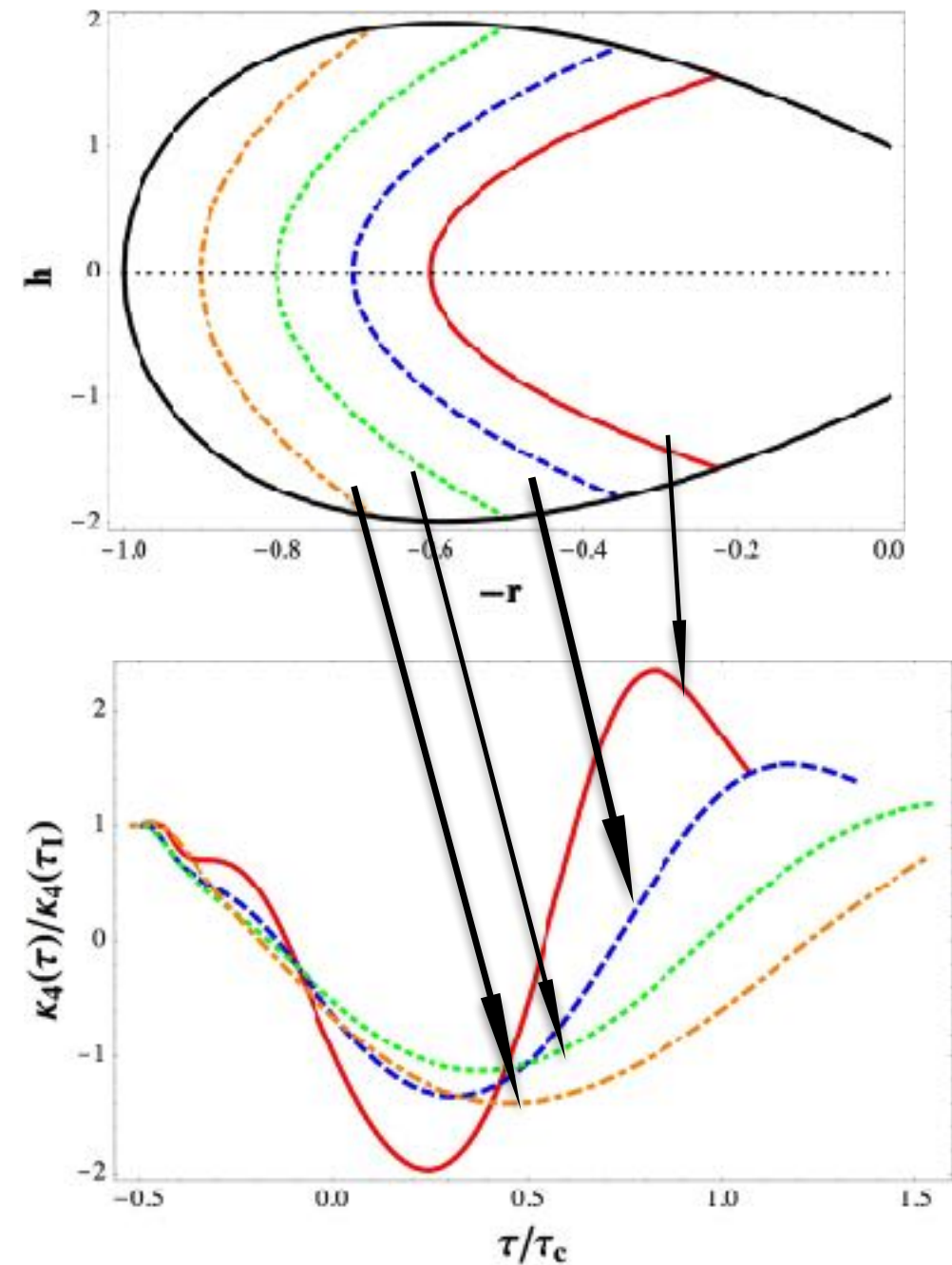


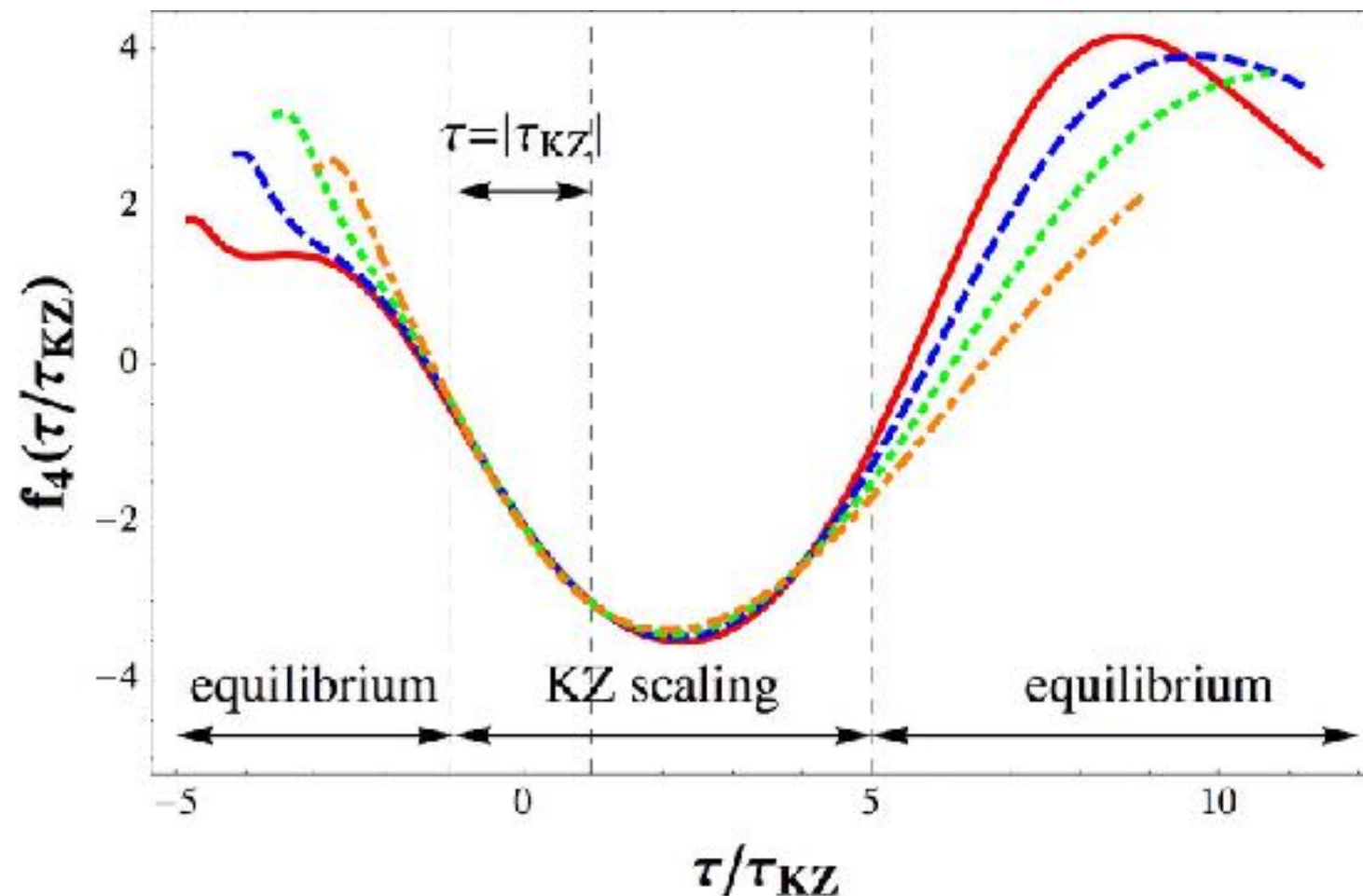
## Difficulties when applied to heavy-ion collisions:

- Extending scaling hypothesis for non-Gaussian cumulants. ✓
- Applying non-equilibrium scaling for trajectories away from the critical point. ✓

S. Mukherjee, R. Venugopalan and YY, PRL,  
Editors' suggestion, 16'

- Ask the key question “what do we expect to see” in a simplified set-up.
- Selecting evolutions with same  $\theta_{KZ}$ .
- Expectation: rescaled evolutions are independent of trajectories.





- Universality regained !

**S. Mukherjee, R. Venugopalan and YY, PRL, Editors' suggestion, 16'**

- Search for KZ scaling in experiment data:

**(Chun Shen, S. Mukherjee, B. Schenke, R. Venugopalan and YY, in progress)**

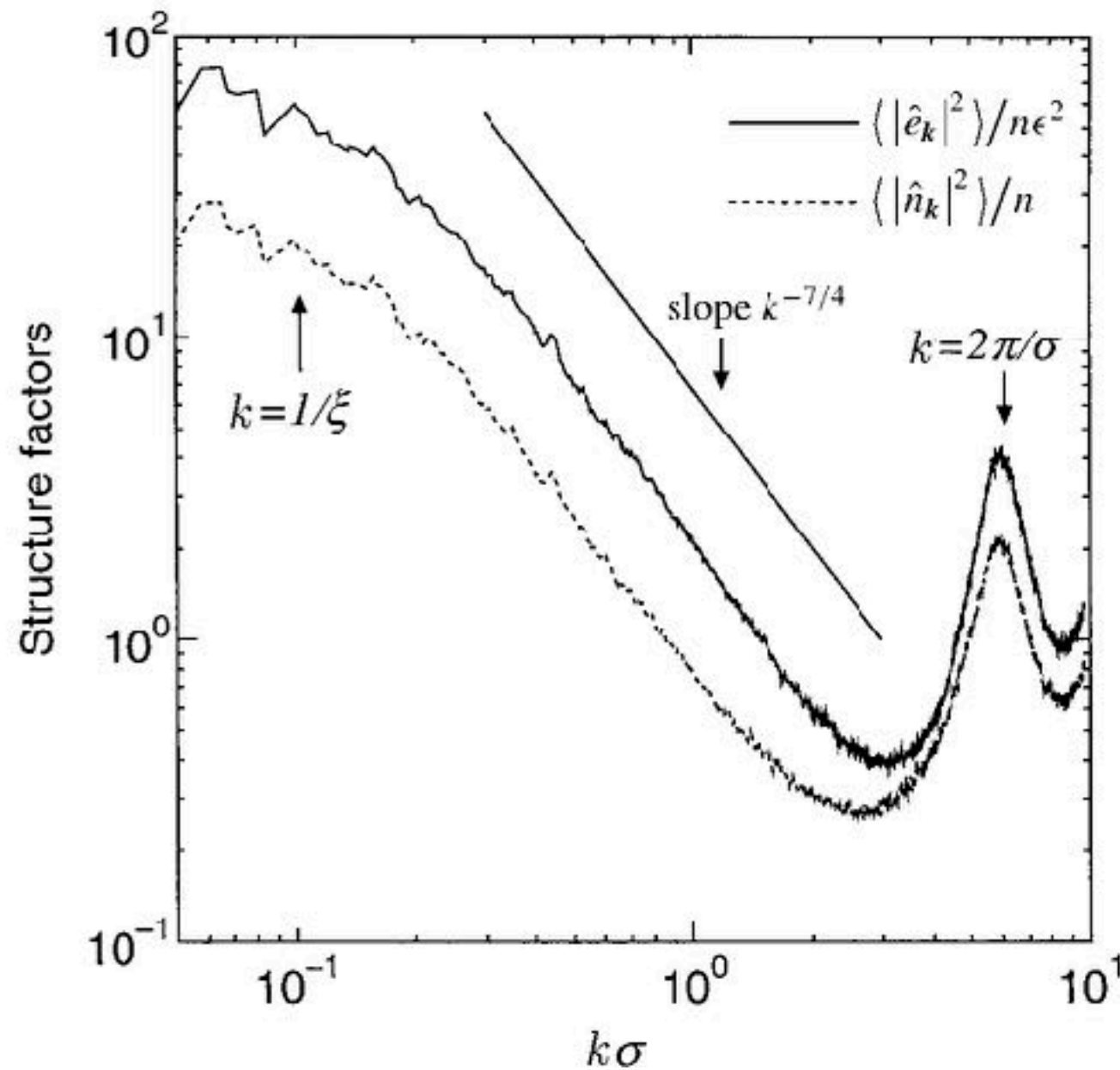
***So far:***

Relaxation rate does not depend on the wave number (gradient).

***Next:***

KZ-dynamics with non-trivial moment-dependence.

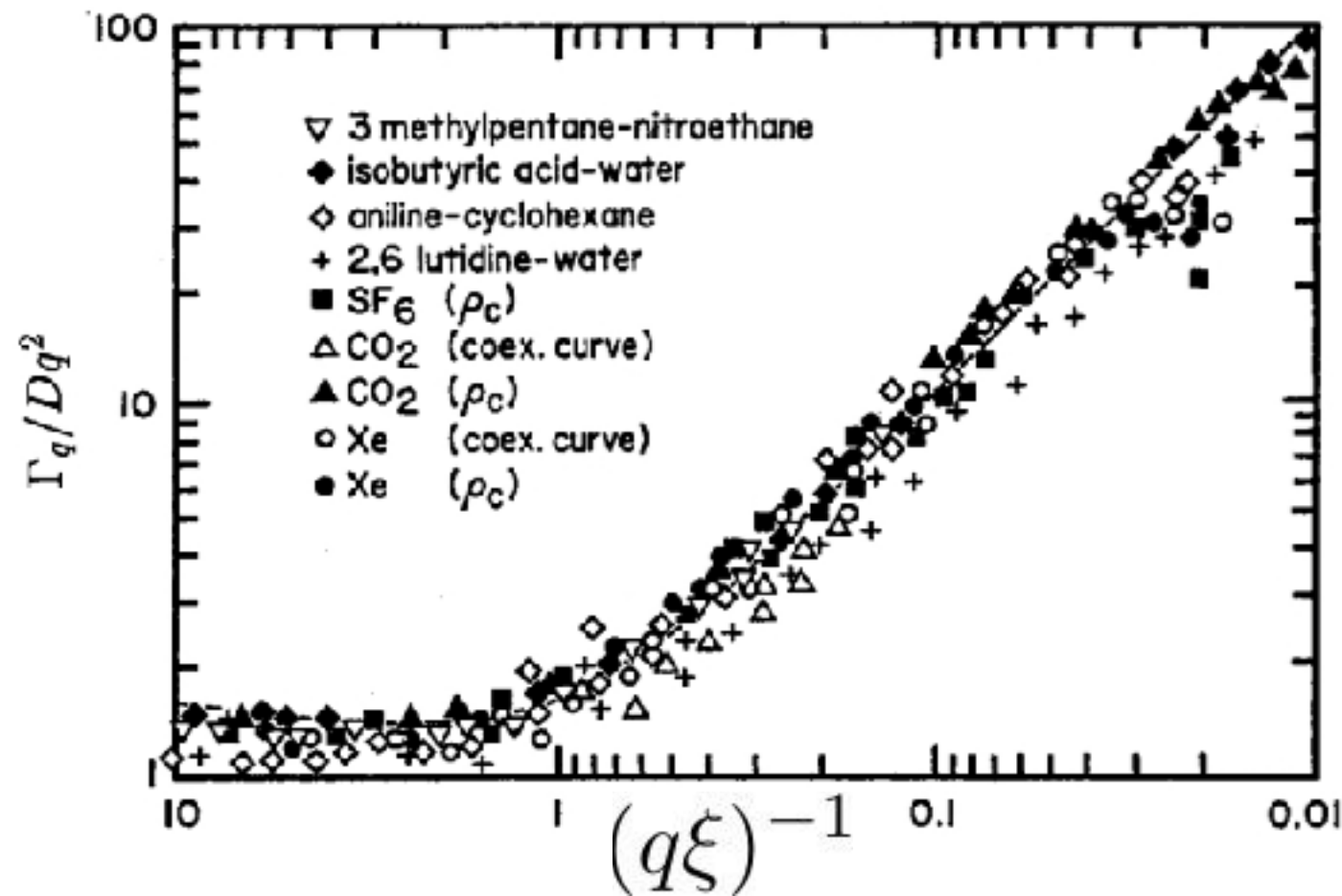
# Momentum-dependence manifests the criticality !



(from Onuki's book)

$$\chi(q) \sim \xi^{2-\eta} f_\chi(q\xi), \quad \begin{aligned} q &\ll \xi^{-1}, & \chi(q) &\sim \xi^{2-\eta} \\ q &\gg \xi^{-1}, & \chi(q) &\sim q^{-(2-\eta)} \end{aligned}$$





(from Onuki's book)

For conserved critical mode, its relaxation time has to be matched to hydrodynamics.

$$\begin{aligned}
 q &\ll \xi^{-1}, & \Gamma(q) &\sim D q^2, \\
 q &\sim \xi^{-1}, & \Gamma(q) &\sim \xi^{-z}, \\
 q &\gg \xi^{-1}, & \Gamma(q) &\sim q^z,
 \end{aligned}$$

# The simplest critical hydrodynamics

(D. Teaney, F. Yan, Y. Akamatsu, YY, in progress)

- Stochastic equation for  $n_B$ :

$$\partial_\tau n_B(\tau, q) = -\Gamma(q) n_B(\tau, q) + \text{noise}$$

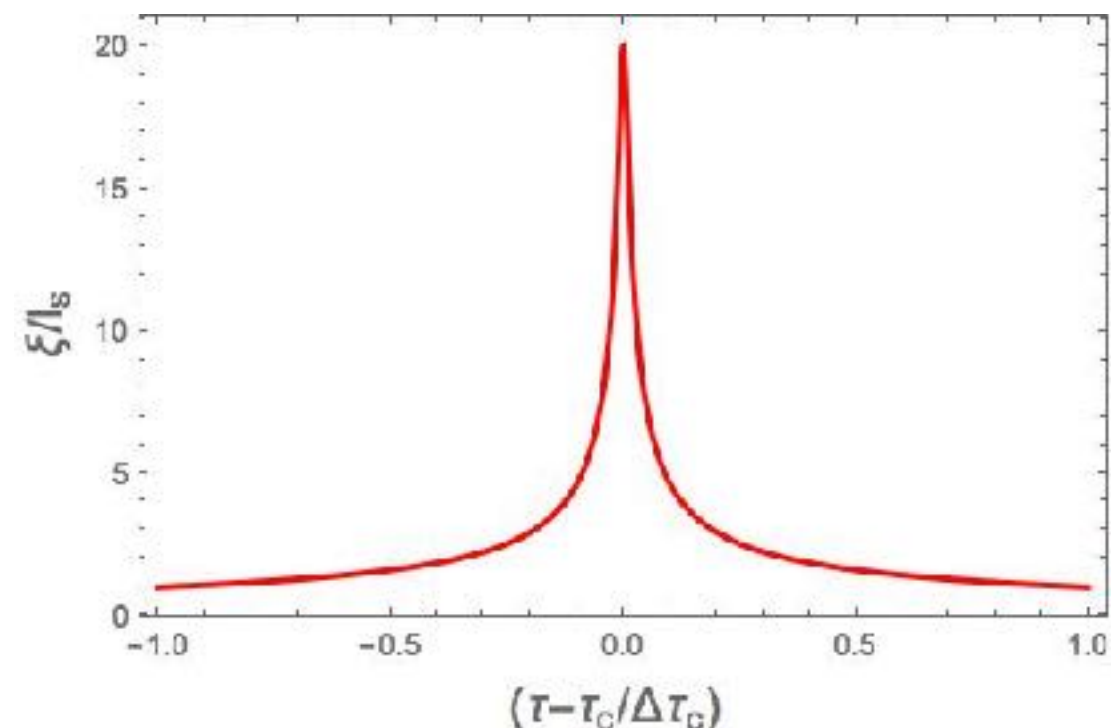


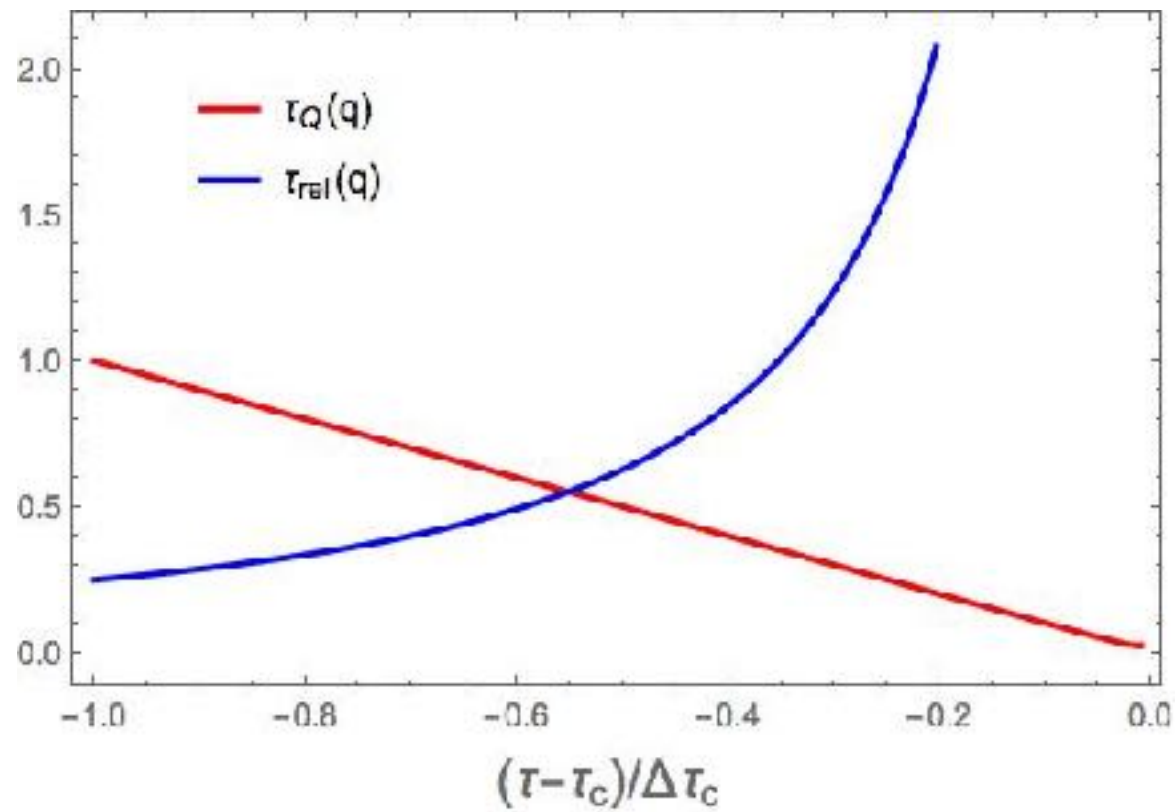
$$\partial_\tau C_{nn}(\tau, q) = -2\Gamma(q) [C_{nn}(\tau, q) - \chi(q)]$$

- System quenches through the critical point (no back-reaction, i.e. Model B)

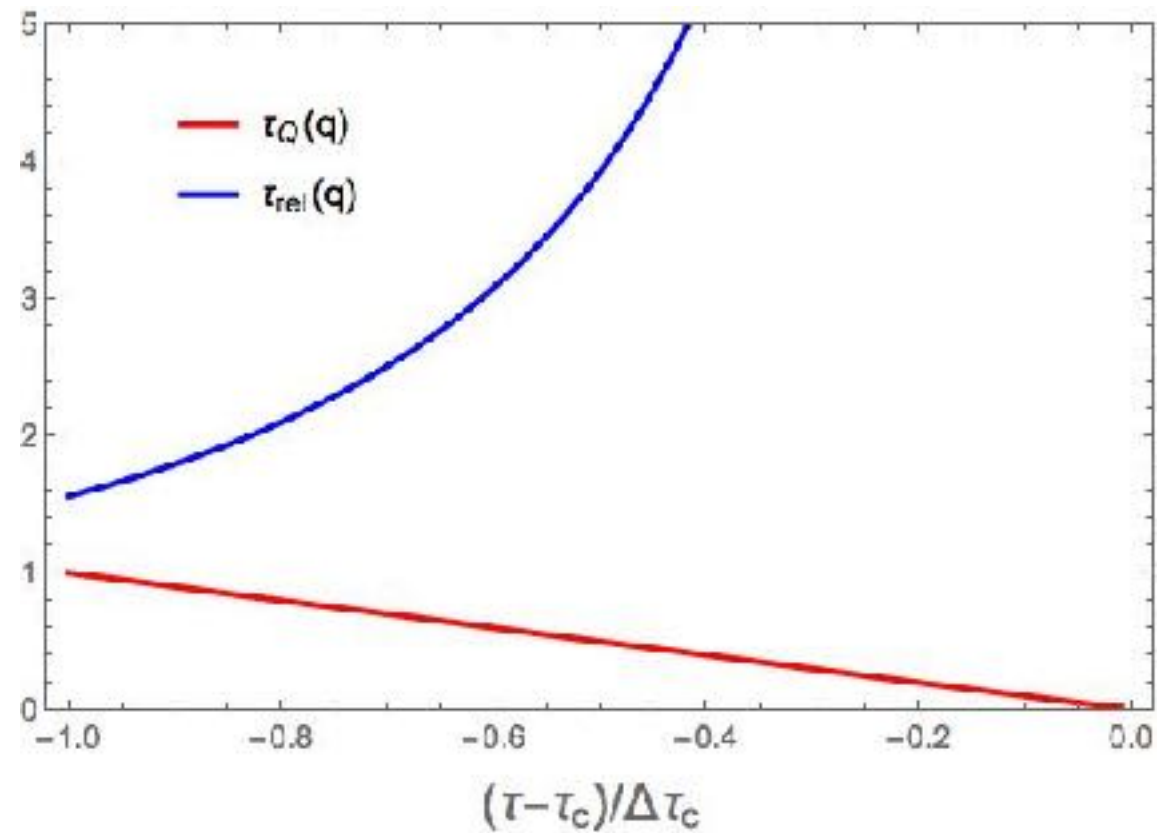
$$\xi(T) \sim \left| \frac{(T - T_c)}{\Delta T_c} \right|^{-\nu}$$

$$T - T_c \sim \left| \frac{(\tau - \tau_c)}{\tau_c} \right|^a$$





$q > q^*$



$q < q^*$

- No KZ dynamics for  $q < q^*$ .

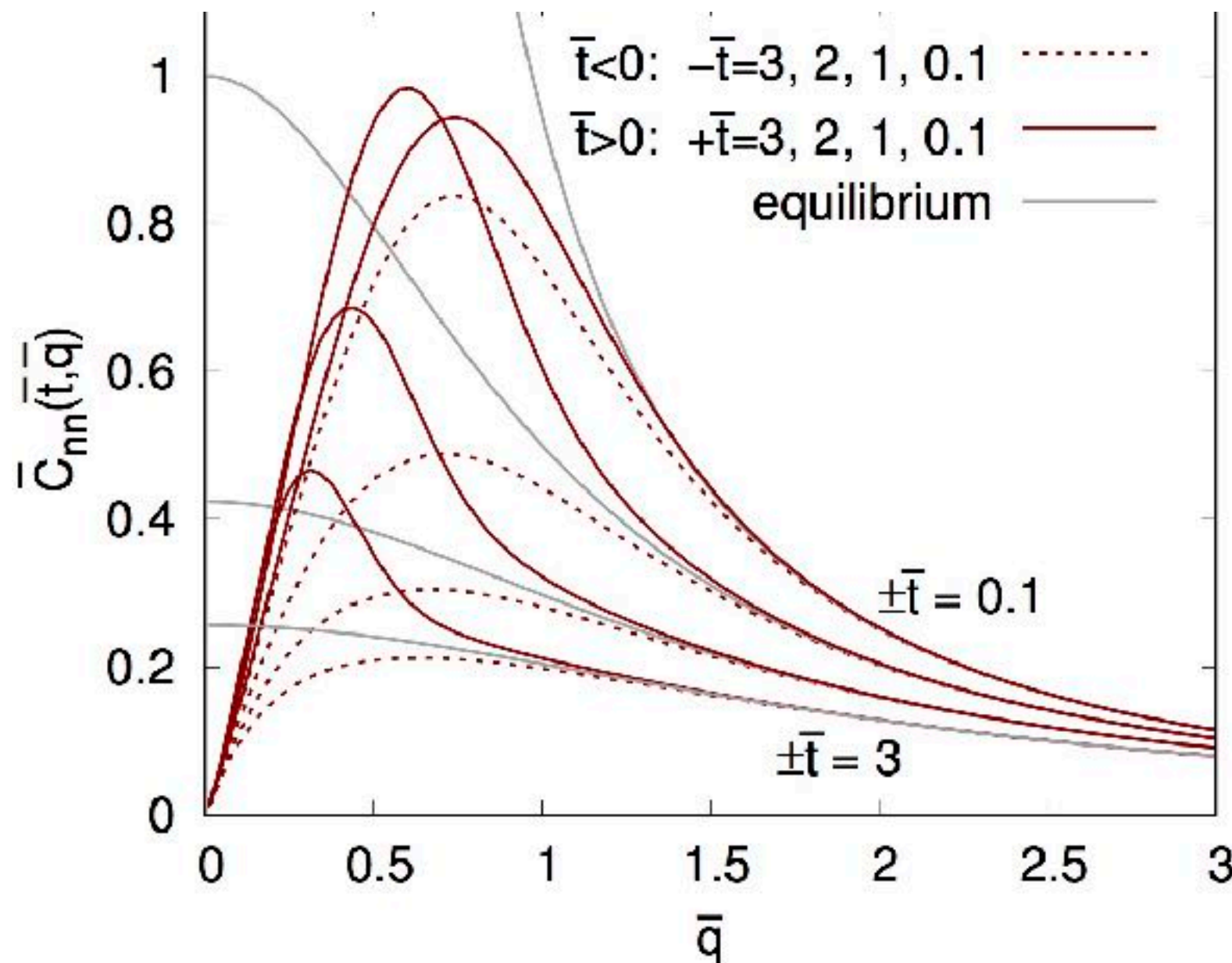
$$q^* \sim \sqrt{\frac{1}{\tau_{\text{mft}} \tau_{\text{exp}}}}$$

(See Yukinao's talk)

# Scaling solution for $q > q^*$

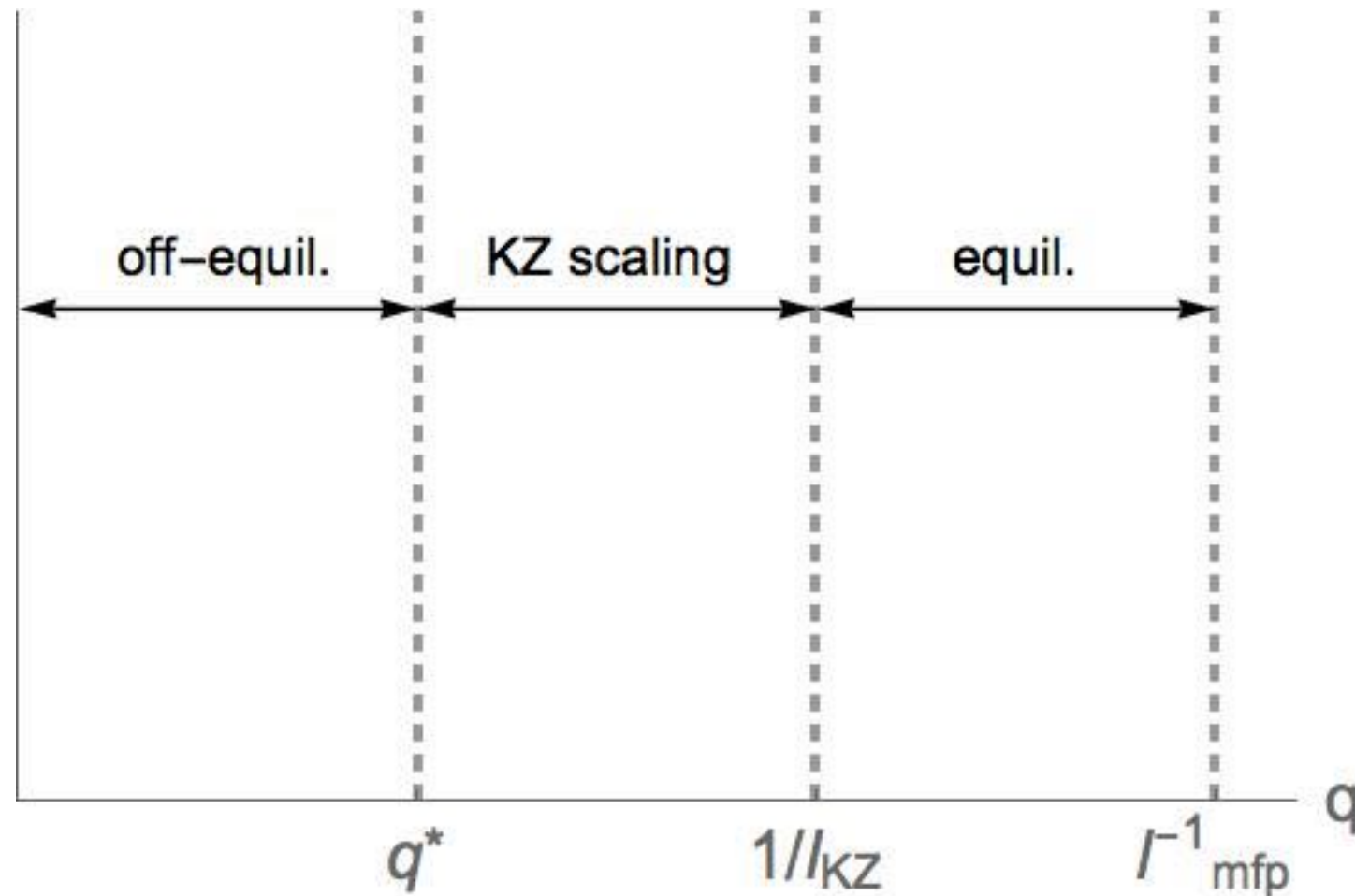
(D. Teaney, F. Yan, Y. Akamatsu, YY, in progress)

$$C_{nn}(\tau, q) \sim l_{KZ}^2 \bar{C}_{nn}(\bar{t}, \bar{q}), \quad \bar{t} \equiv \tau / \tau_{KZ}, \quad \bar{q} \equiv q l_{KZ}.$$



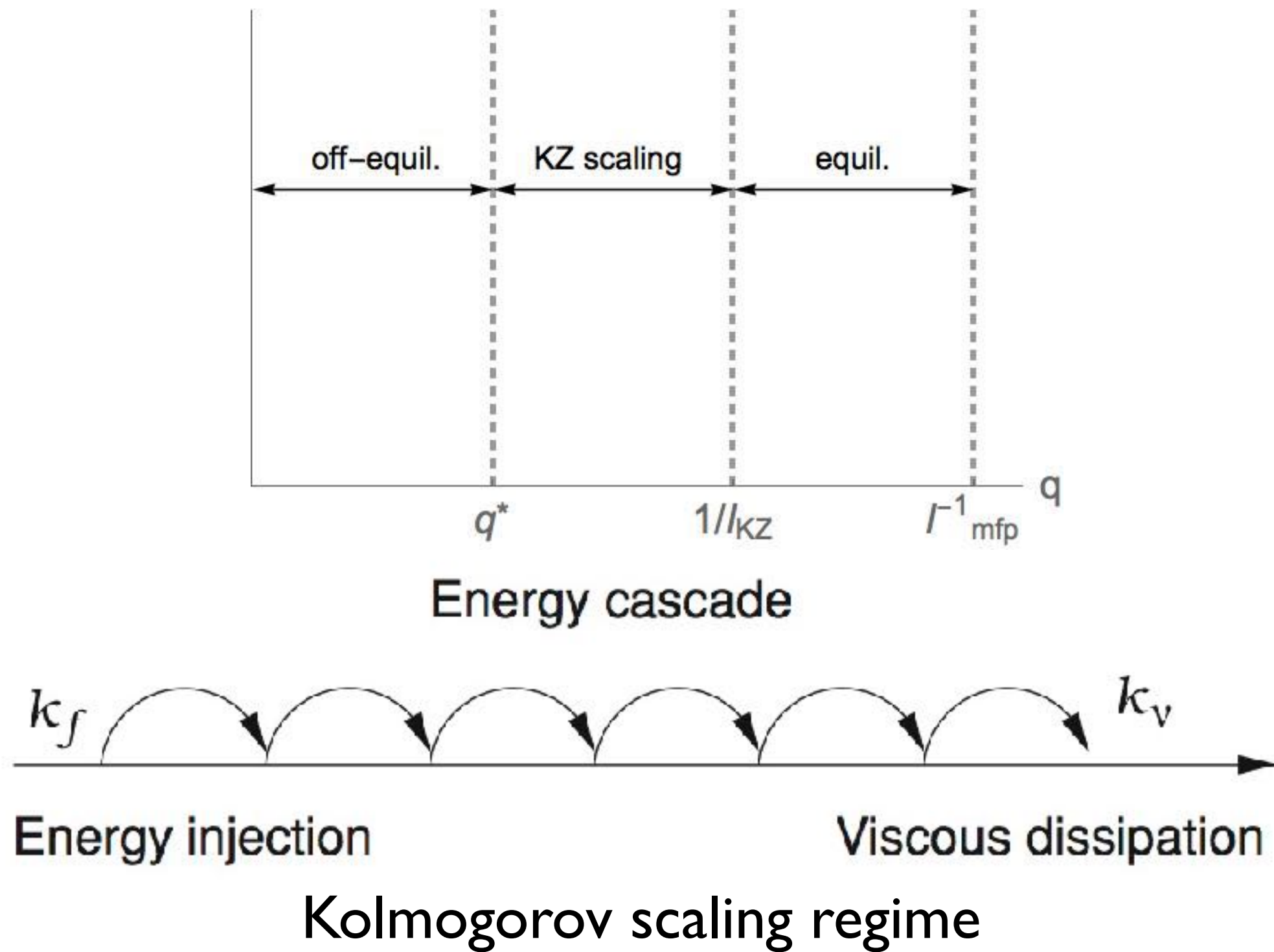
$$\begin{aligned}
 q &\ll \xi^{-1}, & \Gamma(q) &\sim D q^2, \\
 q &\sim \xi^{-1}, & \Gamma(q) &\sim \xi^{-z}, \\
 q &\gg \xi^{-1}, & \Gamma(q) &\sim q^z,
 \end{aligned}$$

# Summary and outlook



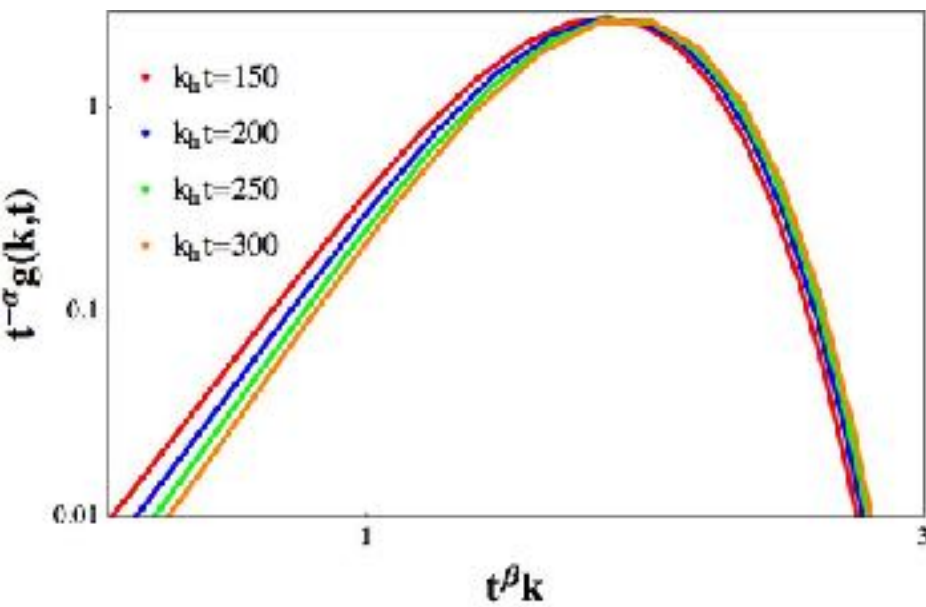
- We have applied KZ dynamics to study QCD critical point.
- Generalization to Critical Hydrodynamics is under way.
- Rich pattern induced by momentum-dependence.



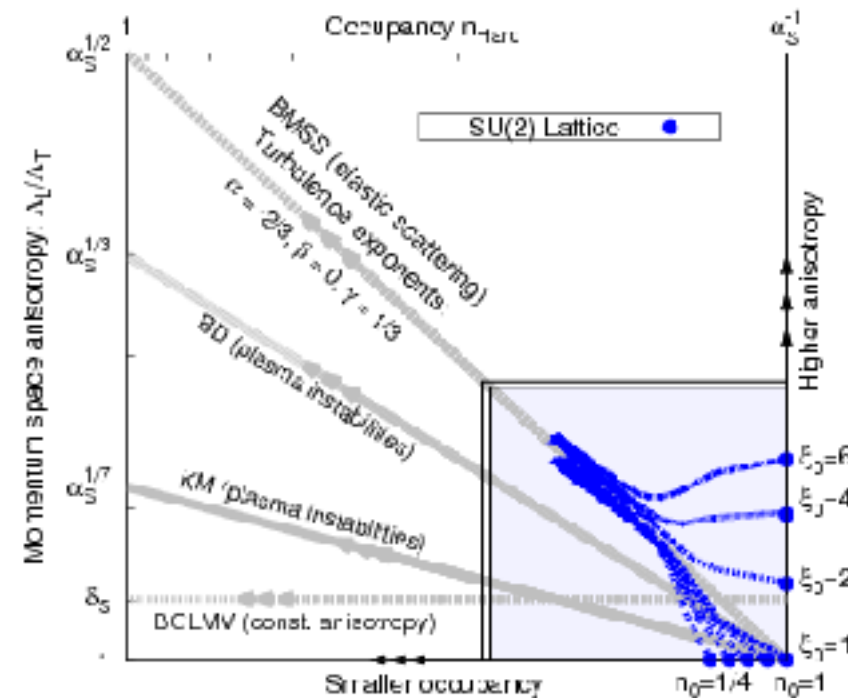


- In analogy to the turbulent cascade.

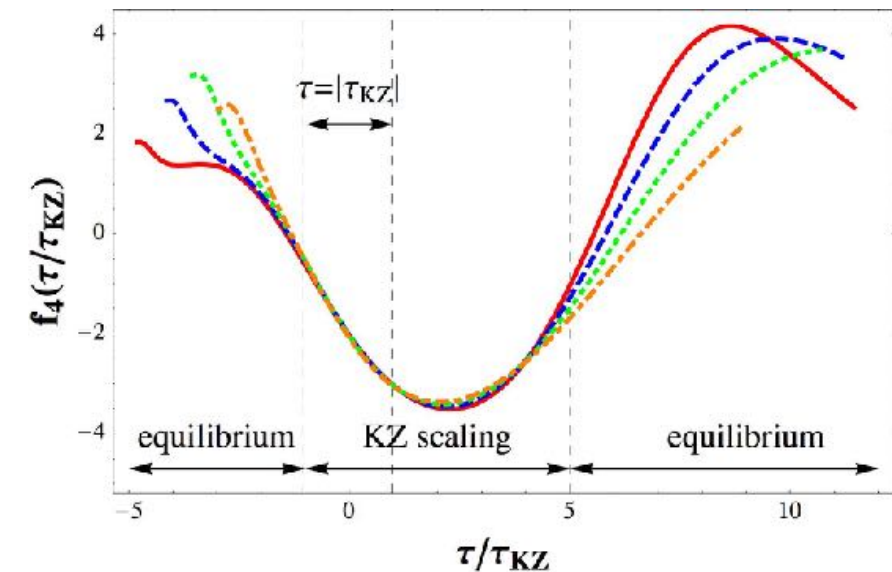
- ▶ Jet quenching and energy loss of high energy probes
- ▶ Heavy quark transports and heavy quarkonia
- ▶ Electromagnetic probes and chiral magnetic effect
- ▶ Thermalization and turbulence
- ▶ QCD critical point



(Y. Hirono, D. Kharzeev and YY, PRD, 15')



(J. Berges, K. Boguslavski and S. Schlichting, R. Venugopalan PRD, 14')



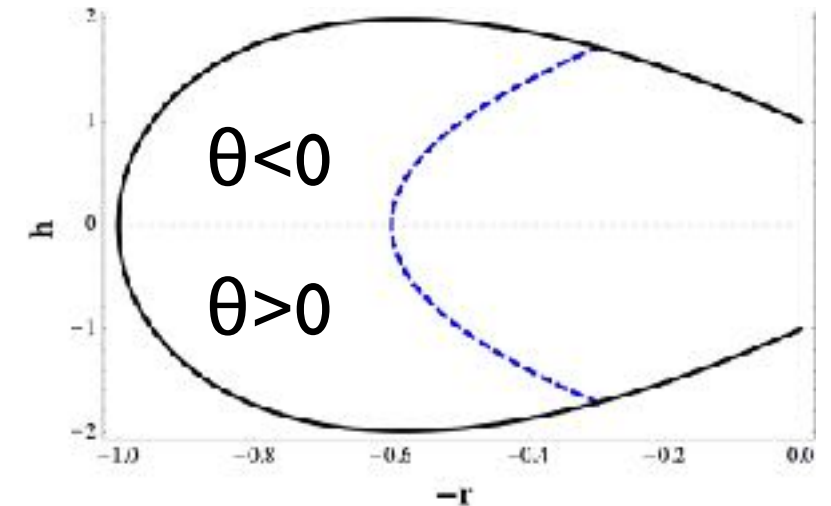
(S. Mukherjee, R. Venugopalan and YY, PRL, Editors' suggestion, 16')

Off-equilibrium scaling and self-similarity: an ubiquitous theme.  
Any unified theory for off-equilibrium dynamics?

# Back-up slides

- Non-Gaussian cumulants depends on  $\xi$  and  $\theta$ .

$$\kappa_n^{\text{eq}} \sim \xi_{\text{eq}}^{\#} \times f_n^{\text{eq}}(\theta)$$



- Additional non-equilibrium scaling variable:  
 (“memory of sign”):  $\theta_{\text{KZ}} = \theta(\tau_{\text{KZ}})$

- Generalized scaling hypothesis:

$$\kappa_n(\tau; \Gamma) \sim (l_{\text{KZ}}(\Gamma))^{\#} f_n(\tau/\tau_{\text{KZ}}; \theta(\tau_{\text{KZ}}))$$

